$$x^2 - y^2 = 24$$
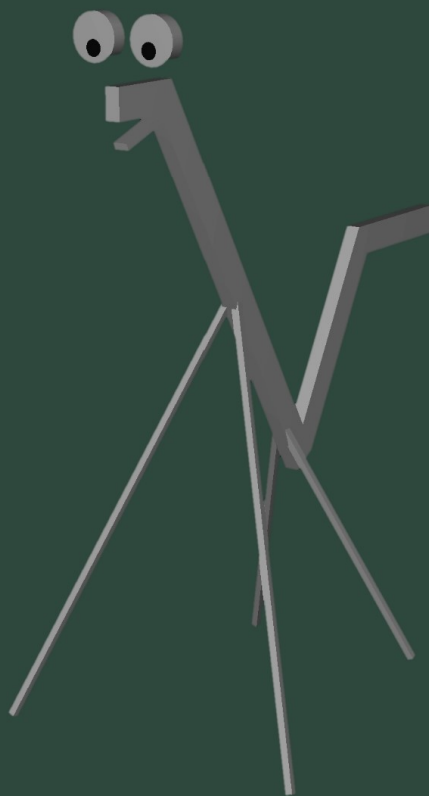$$xy = 35$$
$$x + y = ?$$

$x^2 - y^2 = 24$

$xy = 35$

$x + y = ?$

---< START (the following reasoning requires a calculator) >---

$x^2 - y^2 = 24$

$xy = 35$

---<>---

$x^2 - y^2 = 24$

$(x^2 - y^2)^2 = 24^2$

$x^4 - 2(xy)^2 + y^4 = 576$

because $xy = 35$ then:

$x^4 - 2(35)^2 + y^4 = 576$

$x^4 - 2450 + y^4 = 576$

$x^4 + y^4 = 576 + 2450$

$x^4 + y^4 = 3026$

---<>---

$xy = 35$

$(xy)^4 = 35^4$

$x^4 y^4 = 1500625$

---<>---

$| \ x^4 + y^4 = 3026$
$|$
$| \ x^4 y^4 = 1500625$

let $A = x^4$

let $B = y^4$

$| \ A + B = 3026 \ldots = S$ (as Sum)
$|$
$| \ AB = 1500625 \ldots = P$ (as Product)

formula finding two numbers given their sum (S) and their product (P):

the quadratic equation $k^2 - Sk + P = 0$

$k^2 - 3026k + 1500625 = 0$

$\Delta = -(-3026)^2 - 4*1*1500625 = 9156676 - 6002250 = 3154176$

$\sqrt{\Delta} = \sqrt{3154176} = 1776$

---< root #1: A and x >---

$A = (-(-3026) + 1776)/2*1 = 2401$

$A = x^4 = 2401 \Rightarrow x = \sqrt[4]{2401} = 7$

---< root #2: B and y >---

$B = (-(-3026) - 1776)/2*1 = 625$

$B = y^4 = 625 \Rightarrow y = \sqrt[4]{625} = 5$

---< RESULT(S) >---

$x = 7$

$y = 5$

$x + y = 12$