si

$$x^2 + 5x = -25$$

que vaut

x^3 ?

---------- Q U E S T I O N ----------

si $x^2 + 5x = -25$

que vaut x^3 ?

---------- R É P O N S E (en mode "bourrin+") ----------

$x^2 + 5x = -25$

$x^2 + 5x + 25 = 0$

$\Delta = 5^2 - 4 \cdot 1 \cdot 25 = 25 - 100 = -75$

$\sqrt{\Delta} = \sqrt{(-75)} = i\sqrt{75} = i\sqrt{(3 \cdot 25)} = i\sqrt{(3 \cdot 5^2)} = i\sqrt{(3)} \cdot \sqrt{(5^2)} = 5i\sqrt{3}$

racine #1: $x = (-5 + 5i\sqrt{3})/2 \cdot 1 = -5/2 + (5/2)i\sqrt{3}$

racine #2: $x = (-5 - 5i\sqrt{3})/2 \cdot 1 = -5/2 - (5/2)i\sqrt{3}$

----- rappels -----

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

----- (racine #1)^3 -----

$x = -5/2 + (5/2)i\sqrt{3}$

$x = -2{,}5 + 2{,}5i\sqrt{3}$

$x^3 = (-2{,}5 + 2{,}5i\sqrt{3})^3$

$x^3 = (-2{,}5)^3 + 3 \cdot (-2{,}5)^2 \cdot 2{,}5i\sqrt{3} + 3 \cdot (-2{,}5) \cdot (2{,}5i\sqrt{3})^2 + (2{,}5i\sqrt{3})^3$

$x^3 = -15{,}625 + 81{,}18988i + 140{,}625 - 81{,}18988i$

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+----------+
| x³ = 125 |
+----------+
```

----- (racine #2)^3 -----

$x = -5/2 - (5/2)i\sqrt{3}$

$x = -2{,}5 - 2{,}5i\sqrt{3}$

$x^3 = (-2{,}5 - 2{,}5i\sqrt{3})^3$

$x^3 = (-2{,}5)^3 - 3 \cdot (-2{,}5)^2 \cdot 2{,}5i\sqrt{3} + 3 \cdot (-2{,}5) \cdot (2{,}5i\sqrt{3})^2 - (2{,}5i\sqrt{3})^3$

$x^3 = -15{,}625 - 81{,}18988i + 140{,}625 + 81{,}18988i$

```
+----------+
| x³ = 125 |
+----------+
```

il y a une suite

---------- R É P O N S E (en mode "bourrin-") ----------

$x^2 + 5x = -25$

$x^2 + 5x + 25 = 0$

$\Delta = 5^2 - 4\cdot1\cdot25 = 25 - 100 = -75$

$\sqrt{\Delta} = \sqrt{(-75)} = i\sqrt{75} = i\sqrt{(3\cdot25)} = i\sqrt{(3\cdot5^2)} = i\sqrt{(3)}\cdot\sqrt{(5^2)} = 5i\sqrt{3}$

racine #1: $x = (-5 + 5i\sqrt{3})/2\cdot1 = -5/2 + (5/2)i\sqrt{3}$

racine #2: $x = (-5 - 5i\sqrt{3})/2\cdot1 = -5/2 - (5/2)i\sqrt{3}$

----- rappels -----

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

----- (racine #1)³ -----

$x = -5/2 + (5/2)i\sqrt{3}$

$x = -5/2(1 - i\sqrt{3})$

$x^3 = (-5/2(1 - i\sqrt{3}))^3$

$x^3 = (-5/2)^3\cdot(1 - i\sqrt{3})^3$

$x^3 = -125/8\cdot(1 - i\sqrt{3})^3$

$(1 - i\sqrt{3})^3 = (1 - i\sqrt{3})^2\cdot(1 - i\sqrt{3})$

$(1 - i\sqrt{3})^3 = (1 - 2i\sqrt{3} - 3)\cdot(1 - i\sqrt{3})$

$(1 - i\sqrt{3})^3 = 1 - i\sqrt{3} - 2i\sqrt{3} - 6 - 3 + 3i\sqrt{3}$

$(1 - i\sqrt{3})^3 = 1 - 6 - 3 - 3i\sqrt{3} + 3i\sqrt{3}$

$(1 - i\sqrt{3})^3 = -8$

$x^3 = -125/8\cdot(-8)$

```
+----------+
| x³ = 125 |
+----------+
```

----- (racine #2)³ -----

$x = -5/2 - (5/2)i\sqrt{3}$

$x = -5/2(1 + i\sqrt{3})$

$x^3 = (-5/2(1 + i\sqrt{3}))^3$

$x^3 = (-5/2)^3\cdot(1 + i\sqrt{3})^3$

$x^3 = -125/8 \cdot (1 + i\sqrt{3})^3$

$(1 + i\sqrt{3})^3 = (1 + i\sqrt{3})^2 \cdot (1 + i\sqrt{3})$

$(1 + i\sqrt{3})^3 = (1 + 2i\sqrt{3} - 3) \cdot (1 + i\sqrt{3})$

$(1 + i\sqrt{3})^3 = 1 + i\sqrt{3} + 2i\sqrt{3} - 6 - 3 - 3i\sqrt{3}$

$(1 + i\sqrt{3})^3 = 1 - 6 - 3 + 3i\sqrt{3} - 3i\sqrt{3}$

$(1 + i\sqrt{3})^3 = -8$

$x^3 = -125/8 \cdot (-8)$

```
+----------+
| x^3 = 125 |
+----------+
```

---------- R É P O N S E (en mode "grande classe") ----------

$x^2 + 5x = -25$

$x^2 + 5x + 25 = 0$

$x(x^2 + 5x + 25) = 0$

$x^3 + 5x^2 + 25x = 0$

```
x^3 + 5(x^2 + 5x) = 0
       ---·---
          |
·----------·
|
·--> or x^2 + 5x = -25 (énoncé)
```

donc $x^3 + 5(x^2 + 5x) = 0$ devient:

$x^3 + 5 \cdot (-25) = 0$

$x^3 - 125 = 0$

```
+----------+
| x^3 = 125 |
+----------+
```

----- observation -----

- si $x^3 = 125$ alors $x = \sqrt[3]{125} = 5$
- or si $x = 5$ alors $x^2 + 5x = -25$ devient:
- $5^2 + 5 \cdot 5 = 50$ et non pas -25
- car $x^3 = 125$ est le résultat de 2 nombres complexes

les racines de $x^2 + 5x = -25$ étant:

- $x = -5/2 + (5/2)i\sqrt{3}$
- $x = -5/2 - (5/2)i\sqrt{3}$

et non pas $x = 5$

----- (fin) -----