2 exercices ...

$$3^{(2x - 3)} - 4(3^{(x - 2)}) + 1 = 0$$

$$(x - 6)^4 + (x - 8)^4 = 16$$

... sans calculatrice (pas de logarithmes)

... sans hypothèse "racines évidentes"

---------- Q U E S T I O N ----------

$3^{(2x - 3)} - 4(3^{(x - 2)}) + 1 = 0$

$x = ?$ (dans R)

---------- R É P O N S E ----------

$3^{(2x - 3)} - 4(3^{(x - 2)}) + 1 = 0$

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sachant que:

- $3^{(2x - 3)} = 3^{2x}/3^3 = (1/27)\cdot 3^{2x}$
- $3^{(x - 2)} = 3^x/3^2 = (1/9)\cdot 3^x$
- $4\cdot(1/9)\cdot 3^x = (4/9)\cdot 3^x$

donc $3^{(2x - 3)} - 4(3^{(x - 2)}) + 1 = 0$ devient:

$(1/27)\cdot 3^{2x} - (4/9)\cdot 3^x + 1 = 0$

$(1/27)\cdot 3^x\cdot 3^x - (4/9)\cdot 3^x + 1 = 0$

soit $X = 3^x$

$(1/27)\cdot X\cdot X - (4/9)\cdot X + 1 = 0$

$(1/27)\cdot X^2 - (4/9)\cdot X + 1 = 0$

$\Delta = (-4/9)^2 - 4\cdot(1/27)\cdot 1 = 16/81 - 4/27 = 16/81 - 12/81 = 4/81$

$\sqrt{\Delta} = \sqrt{(4/81)} = +2/9$ et $-2/9$

- cas $\sqrt{\Delta} = +2/9$: $X = (-(-4/9) + 2/9)/2\cdot(1/27) = (6/9)/(2/27) = 9$
- cas $\sqrt{\Delta} = -2/9$: $X = (-(-4/9) - 2/9)/2\cdot(1/27) = (2/9)/(2/27) = 3$

$X = 3^x \Rightarrow 9 = 3^x \Rightarrow x = 2$

$X = 3^x \Rightarrow 3 = 3^x \Rightarrow x = 1$

$\boxed{x = 2}$ et $\boxed{x = 1}$

---------- Q U E S T I O N ----------

$(x - 6)^4 + (x - 8)^4 = 16$

$x = ?$ (dans R et C)

---------- R É P O N S E ----------

$(x - 6)^4 + (x - 8)^4 = 16$

$(x - 6)^4 + (x - 6 - 2)^4 = 16$

soit $x - 6 = y$

si $x - 6 = y$ alors $(x - 6)^4 + (x - 6 - 2)^4 = 16$ devient:

$y^4 + (y - 2)^4 = 16$

rappel: $(a - b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

$y^4 + (y^4 - 8y^3 + 24y^2 - 32y + 16) = 16$

$y^4 + y^4 - 8y^3 + 24y^2 - 32y + 16 - 16 = 0$

$2y^4 - 8y^3 + 24y^2 - 32y = 0$

$2y \cdot (y^3 - 4y^2 + 12y - 16) = 0$

$2y = 0$

$y = 0$

$x - 6 = y$

$x - 6 = 0$

$\boxed{x = 6}$ <--- racine #1

----- pour résoudre $y^3 - 4y^2 + 12y - 16 = 0$: méthode Cardan/Tartaglia -----

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$y^3 - 4y^2 + 12y - 16 = 0$

(voir page suivante)

----- étape 1 sur 2: annulation du terme de second degré ($4y^2$) -----

$a = 1;\ b = -4;\ c = 12;\ d = -16$

$y = k - b/3a$

$y = k - (-4)/(3\cdot 1)$

$y = k + 4/3$

if $y = k + 4/3$ then $y^3 - 4y^2 + 12y - 16 = 0$ becomes:

$(k + 4/3)^3 - 4(k + 4/3)^2 + 12(k + 4/3) - 16 = 0$

$(k^3 + 4x^2 + 16k/3 + 64/27) - (4k^2 + 32k/3 + 64/9) + (12k + 48/3) - 16 = 0$

$k^3 + 4x^2 + 16k/3 + 64/27 - 4k^2 - 32k/3 - 64/9 + 12k + 48/3 - 16 = 0$

$k^3 + 4x^2 - 4k^2 + 16k/3 - 32k/3 + 12k + 64/27 - 64/9 + 48/3 - 16 = 0$

$k^3 + 16k/3 - 32k/3 + 36k/3 + 64/27 - 192/27 + 432/27 - 432/27 = 0$

$k^3 + 20k/3 - 128/27 = 0$

$k^3 + 20k/3 - 128/27 = 0$ est basé sur le modèle $k^3 + pk + q = 0$ avec:

$p = 20/3$

$q = -128/27$

----- étape 2 sur 2: application de la méthode de C/T -----

$k = [-q/2 + \sqrt{(q^2/4 + p^3/27)}]^{(1/3)} + [-q/2 - \sqrt{(q^2/4 + p^3/27)}]^{(1/3)}$

$k = [-(-128/27)/2 + \sqrt{((-128/27)^2/4 + (20/3)^3/27)}]^{(1/3)} +$

$\quad [-(-128/27)/2 - \sqrt{((-128/27)^2/4 + (20/3)^3/27)}]^{(1/3)}$

$k = 0{,}66666... = 2/3$

$y = k + 4/3 = 2/3 + 4/3 = 6/3 = 2$

$x - 6 = y$

$x - 6 = 2$

$x = 2 + 6$

(voir page suivante)

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|   x = 8   | <--- racine #2
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----- les autres racines (dans C) -----
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----- division euclidienne entre y³ - 4y² + 12y - 16 et y - 2 -----

(y³ - 4y² + 12y - 16)/(y - 2) = y² - 2y + 8

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y² - 2y + 8 = 0

Δ = (-2)² - 4·1·8 = 4 - 32 = -28 = -2²·7

√Δ = +2i√7 et -2i√7

• cas √Δ = +2i√7: y = (-(-2) + 2i√7)/2·1 = 2/2 + 2i√7/2 = 1 + i√7

• cas √Δ = -2i√7: y = (-(-2) - 2i√7)/2·1 = 2/2 - 2i√7/2 = 1 - i√7

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x - 6 = y

x - 6 = 1 + i√7

x = 1 + i√7 + 6

x = 7 + i√7

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x - 6 = y

x - 6 = 1 - i√7

x = 1 - i√7 + 6

x = 7 - i√7

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|   x = 7 + i√7   |
 -----------------  <--- racines #3 et #4
|   x = 7 - i√7   |
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