$$x + 1/y = 2$$

$$y + 1/z = 3$$

$$z + 1/x = 4$$

le produit $xyz = ?$

et procéder à la vérification

$$\begin{cases} x + 1/y = 2 \\ y + 1/z = 3 \\ z + 1/x = 4 \end{cases}$$

$xyz = ?$

$$\begin{cases} x + 1/y = 2 \\ y + 1/z = 3 \\ z + 1/x = 4 \end{cases}$$

$(x + 1/y) \cdot (y + 1/z) \cdot (z + 1/x) = 24$

$(xy + x/z + 1 + 1/yz) \cdot (z + 1/x) = 24$

$xyz + y + x + 1/z + z + 1/x + 1/y + 1/xyz = 24$

$xyz + 1/xyz + (x + 1/y) + (y + 1/z) + (z + 1/x) = 24$

note: comme $x + 1/y = 2$, $y + 1/z = 3$ et $z + 1/x = 4$ (données de l'énoncé)

   alors $xyz + 1/xyz + (x + 1/y) + (y + 1/z) + (z + 1/x) = 24$ devient:

$xyz + 1/xyz + 2 + 3 + 4 = 24$

$xyz + 1/xyz = 15$

soit $k = xyz$

$k + 1/k = 15$

$k \cdot (k + 1/k) = 15k$

$k^2 + k/k = 15k$

$k^2 - 15k + 1 = 0$

$\Delta = (-15)^2 - 4 \cdot 1 \cdot 1 = 225 - 4 = 221$

$\sqrt{\Delta} = \pm\sqrt{221}$

- racine #1: k = (-(-15) + √221)/2·1 = (15 + √221)/2 ≈ 14,933034

- racine #2: k = (-(-15) - √221)/2·1 = (15 - √221)/2 ≈ 0,066965

rappel: xyz = k

```
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|   xyz = (15 ± √221)/2   |
--------------------------
```

```
------------------------
----- vérification -----
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```

```
| L1: x + 1/y = 2
|
| L2: y + 1/z = 3
|
| L3: z + 1/x = 4
```

----- calcul de z -----

L2: y + 1/z = 3 => y = 3 - 1/z

L3: z + 1/x = 4 => 1/x = 4 - z => x = 1/(4 - z)

si y = 3 - 1/z et x = 1/(4 - z) alors L1 devient:

1/(4 - z) + 1/(3 - 1/z) = 2

(3 - 1/z)/((4 - z)·(3 - 1/z)) + (4 - z)/((4 - z)·(3 - 1/z)) = 2

(3 - 1/z)/(12 - 4/z - 3z + 1) + (4 - z)/(12 - 4/z - 3z + 1) = 2

(3 - 1/z)/(13 - 4/z - 3z) + (4 - z)/(13 - 4/z - 3z) = 2

((3 - 1/z) + (4 - z))/(13 - 4/z - 3z) = 2

(3 - 1/z + 4 - z)/(13 - 4/z - 3z) = 2

(7 - 1/z - z)/(13 - 4/z - 3z) = 2

7 - 1/z - z = 2·(13 - 4/z - 3z)

7 - 1/z - z = 26 - 8/z - 6z

8/z - 1/z + 6z - z = 26 - 7

7/z + 5z = 19

7/z + 5z²/z = 19

$(5z^2 + 7)/z = 19$

$5z^2 + 7 = 19z$

$5z^2 - 19z + 7 = 0$

$\Delta = (-19)^2 - 4\cdot5\cdot7 = 221$

$\sqrt{\Delta} = \pm\sqrt{221}$

• racine #1: $z = (-(-19) + \sqrt{221})/2\cdot5 = (19 + \sqrt{221})/10 \approx 3{,}386606$

• racine #2: $z = (-(-19) - \sqrt{221})/2\cdot5 = (19 - \sqrt{221})/10 \approx 0{,}413393$

```
 -----------------------
|   z = (19 ± √221)/10   |
 -----------------------
```

----- calcul de y -----

L2: $y + 1/z = 3$

$y + 1/[(19 + \sqrt{221})/10] = 3$

$y = 3 - 1/[(19 + \sqrt{221})/10]$

$y = 3 - 10/(19 + \sqrt{221})$

$y = 3 - [10\cdot(19 - \sqrt{221})]/[(19 + \sqrt{221})\cdot(19 - \sqrt{221})]$

$y = 3 - [10\cdot(19 - \sqrt{221})]/[(19^2 - (\sqrt{221})^2)]$

$y = 3 - [10\cdot(19 - \sqrt{221})]/[361 - 221]$

$y = 3 - [10\cdot(19 - \sqrt{221})]/140$

$y = 3 - (19 - \sqrt{221})/14$

$y = 42/14 - (19 - \sqrt{221})/14$

$y = [42 - 19 + \sqrt{221}]/14$

$y = (23 + \sqrt{221})/14$

note: même raisonnement avec $z = (19 - \sqrt{221})/10$

$y = (23 - \sqrt{221})/14$

```
 -----------------------
|   y = (23 ± √221)/14   |
 -----------------------
```

```
----- calcul de x -----

L1: x + 1/y = 2

x + 1/[(23 + √221)/14] = 2

x = 2 - 1/[23 + √221)/14]

x = 2 - 14/(23 + √221)

x = 2 - [14·(23 - √221)]/[(23 + √221)·(23 - √221)]

x = 2 - [14·(23 - √221)]/[(23² - (√221)²)]

x = 2 - [14·(23 - √221)]/[529 - 221]

x = 2 - [14·(23 - √221)]/308

x = 2 - (23 - √221)/22

x = 44/22 - (23 - √221)/22

x = [44 - 23 + √221]/22

x = (21 + √221)/22

note: même raisonnement avec y = (23 - √221)/14

x = (21 - √221)/22

 ------------------------
|   x = (21 ± √221)/22   |
 ------------------------

----- calul de xyz -----

xyz = [(21 + √221)/22]·[(23 + √221)/14]·[(19 + √221)/10]

xyz ≈ 14,933034 <--- ok

xyz = [(21 - √221)/22]·[(23 - √221)/14]·[(19 - √221)/10]

xyz ≈ 0,066965 <--- ok

(fin)
```