Quel raisonnement

$$\sqrt{1 + \sqrt{1 + a}} = \sqrt[3]{a}$$

pour trouver a ?

(dans R)

SOLUTION

$\sqrt{[1 + \sqrt{(1 + a)}]} = \sqrt[3]{a}$

a = ?
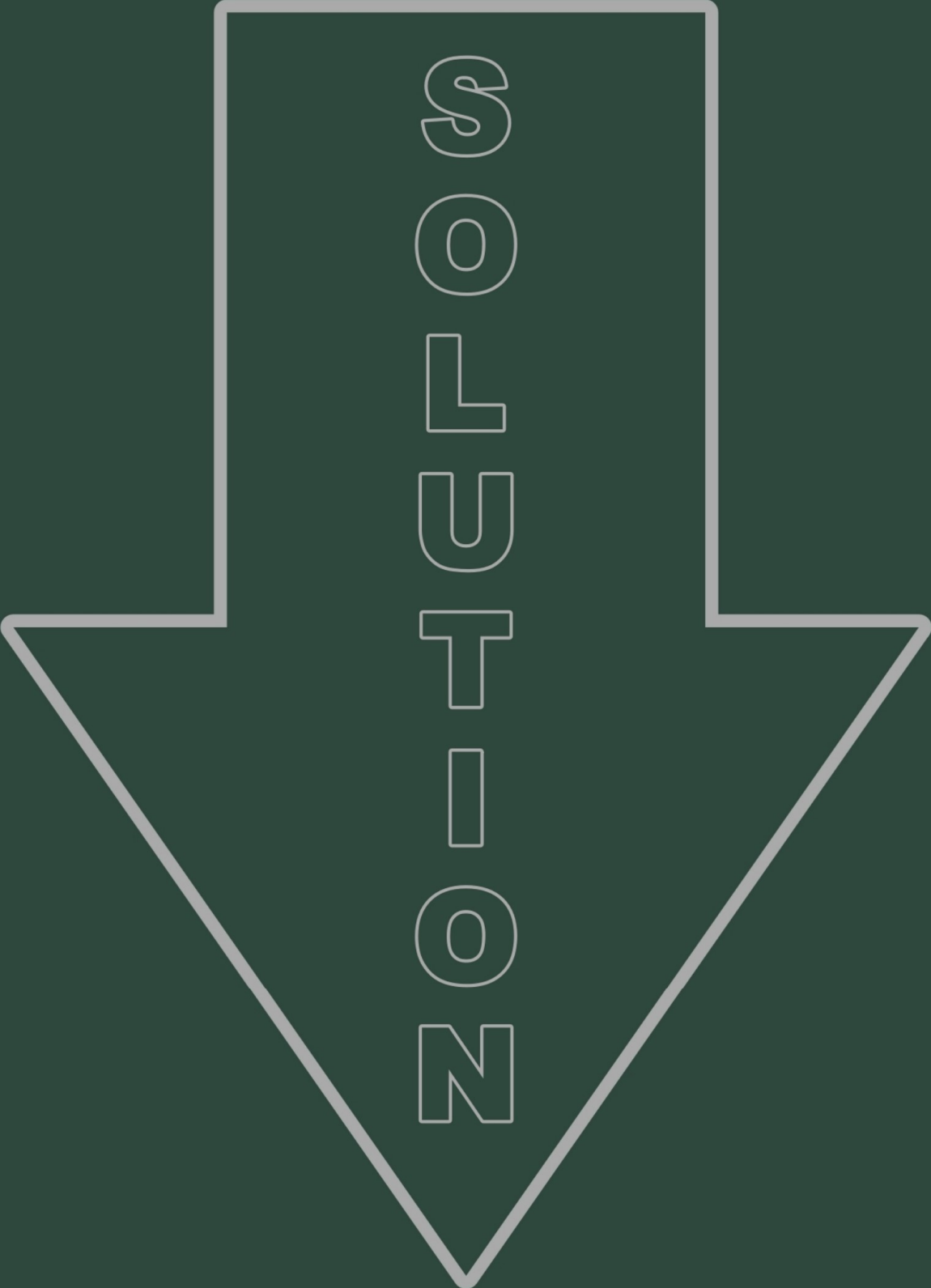
$\sqrt{[1 + \sqrt{(1 + a)}]} = \sqrt[3]{a}$

$1 + \sqrt{(1 + a)} = a^{(2/3)}$

$\sqrt{(1 + a)} = a^{(2/3)} - 1$

$1 + a = (a^{(2/3)} - 1)^2$

$1 + a = a^{(4/3)} - 2 \cdot a^{(2/3)} + 1$

$a^{(4/3)} - 2 \cdot a^{(2/3)} + 1 - 1 - a = 0$

$a^{(4/3)} - 2 \cdot a^{(2/3)} - a = 0$

$[a^{(4/3)} - 2 \cdot a^{(2/3)} - a]/a = 0$

$a^{(1/3)} - 2 \cdot a^{(-1/3)} - 1 = 0$

$a^{(1/3)} - 2 \cdot (1/a^{(1/3)}) - 1 = 0$

$a^{(1/3)} - 2/a^{(1/3)} - 1 = 0$

soit $k = a^{(1/3)}$

$k - 2/k - 1 = 0$

$k^2/k - 2/k - k/k = 0$

$(k^2 - 2 - k)/k = 0$

$k^2 - 2 - k = 0$

$k^2 - k - 2 = 0$

$\Delta = (-1)^2 - 4 \cdot 1 \cdot (-2) = 1 + 8 = 9$

√Δ = ±√9 = ±3

- racine #1: k = (-(-1) + 3)/2·1 = (1 + 3)/2 = 4/2 = 2
- racine #2: k = (-(-1) - 3)/2·1 = (1 - 3)/2 = -2/2 = -1

----- racine #1 -----

a^(1/3) = k = 2

a^(1/3) = 2

a = 2^3

a = 8

contrôle:

- √[1 + √(1 + 8)] = ³√8
- 2 = 2 <-- correct !

----- racine #2 -----

a^(1/3) = k = -1

a^(1/3) = -1

a = -1^3

a = -1

contrôle:

- √[1 + √(1 + (-1))] = ³√(-1)
- 1 = -1 <-- incorrect (racine rejetée) !

----- résultat final -----

■ a = 8